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WAVELET-HILBERT TRANSFORM-BASED APPROACH FOR BUILDING CAPACITY CURVE ESTIMATION IN STRUCTURAL HEALTH MONITORING

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ABSTRACT

Signal processing is essential in Structural Health Monitoring (SHM) for accurate building damage assessment. Previous studies have addressed this by estimating the building capacity curve by reducing Multi Degree of Freedom (MDOF) system to an equivalent Single Degree of Freedom (SDOF) system. However, challenges remain in correctly isolating the fundamental vibration mode while neglecting higher mode contributions to obtain the equivalent SDOF response. Based on this issue, the present study synergistically combines the Discrete Wavelet Transform (DWT) and the Hilbert Transform (HT) to performs a dynamic filtering process in both time and frequency domains, enhancing the isolation of the fundamental vibration mode. This new approach is validated with numerical simulations from several Reinforced Concrete (RC) frame buildings models subjected to different seismic scenarios for different levels of non-linearity. Results indicate that the proposed approach significantly improves the isolation of the fundamental vibration mode compared to previous research, although it has minimal impact on the final capacity curve estimation in most cases. However, there is a substantial improvement in the estimated hysteretic response curves, offering a significant advantage in producing more accurate hysteretic models for constructing reliable mathematical models. Further research is recommended to explore scenarios with significant impact on the estimated capacity curve, considering capturing the progression of structural damage over time during seismic events. Additionally, algorithms for the automatic definition of the boundary frequencies for the dynamic filter process should be developed to reduce the subjective judgment and its practical application in SHM systems.

Keywords: RC building, DWT, HT, Capacity curve, SHM.

1. INTRODUCTION

After a catastrophic earthquake occurs, it is essential to evaluate the damage in buildings to prevent human losses by the collapse of these structures against the possibility of an aftershock (Kusunoki, 2017). This urgency has led to a global increase in the implementation of SHM systems, which combine data acquisition systems and signal processing techniques on acceleration records that represent the dynamic response of buildings subjected to seismic ground excitations, ensuring the denoising and preserving the structural behavior with the aim of building damage assessment. However, significant challenges arise when analyzing these kinds of signals where conventional filtering approaches, based on the obtention of displacements from double integration of acceleration record, often neglect low-frequency components to mitigate integration errors while inadvertently eliminating information about residual displacements, making it challenging to get an accurate damage estimation. Despite these challenges, some studies based on the DWT, an effective technique for signal filtering through their decomposition

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into different frequency bands, have shown remarkable progress (Kusunoki et al., 2018). This methodology isolates the first vibration mode from the total response of a building by reduction of its MDOF system to an equivalent SDOF system, with the aim of estimating the building capacity curve. In this sense, the DWT works like a frequency-invariant filter meanly when the first mode vibration frequency is separated enough from the others. Limitations arise when a high grade of non-linearity causes the second mode frequency to converge with the first frequency mode zone associated with prior elastic response. In such scenarios, a frequency-invariant filter is no longer adequate. The HT is another technique used in signal processing, which generates analytical signals from which instantaneous phases, frequencies, and amplitudes can be calculated to represent them in a frequency-time-energy distribution, which provides another perspective to develop a dynamic-frequency filter process. Based on the abovementioned, this study proposes synergistically combining the strengths of the DWT and HT to develop a dynamic-frequency filter process for the improvement of the first mode isolation for its applicability in the capacity curve estimation for buildings subjected to seismic excitations.

2. DATA

For this study, scenarios where structural damage during a mainshock sequence causes the second mode frequency to approach the first mode (time-varying contribution of ranks), a diverse array of nonlinear RC frame models (two-, six-, twelve-, and twenty-story) with a floor height of 3.0 m and uniform mass distribution were developed based on a simplified model (Figure 1) that accurately represents the shear frame inelastic behavior and using the two-phase Japanese Building Standard Law for seismic design addressing the "allowable stress" and "ultimate capacity" criteria.

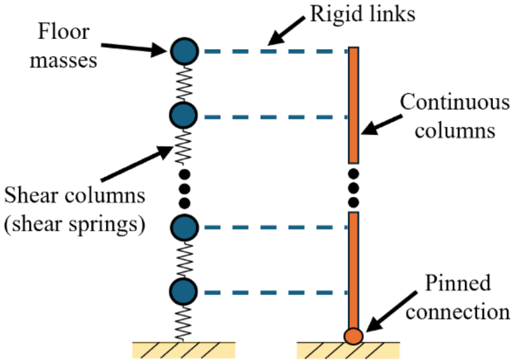

Figure 1. Simplified model

Time history analyses were performed using OpenSEES based on the codes provided by Dr. Trevor Yeow developed in (Yeow & Kusunoki, 2022). The hysteretic behavior of shear spring was modeled using the "Pinching4 Material." A Rayleigh damping coefficient of 5% ratio was applied to the first mode period. The resulting modal period and frequencies for each model are shown in Table 1.

Table 1. Modal properties of RC building models.

	Period [s] (Frequency [Hz])		
Building model	**Mode 1**	**Mode 2**	**Mode 3**
02-story	0.12 (8.42)	0.05 (20.0)	-
06-story	0.36 (2.79)	0.12 (8.37)	0.07 (14.1)
12-story	0.72 (1.39)	0.24 (4.21)	0.14 (7.12)
20-story	1.20 (0.83)	0.46 (2.16)	0.29 (3.50)

For each model, several time history analyses were performed (24 cases) under six scaled single horizontal component seismic records. The records were selected from the PEER database, which were also analyzed in (Yeow & Kusunoki, 2022) for cases where the time-varying contribution of ranks occurs. Each time history analysis was performed with a time-step of 0.001s; then the acceleration responses were resampled to a time-step of 0.01s and contaminated with a white noise excitation of Gaussian distribution utilizing a standard deviation of 0.003G to simulate actual sensor recording noise to the application of the proposed methodology.

3. METHODOLOGY

This hybrid methodology combines the DWT and the HT to improve the first vibration mode isolation from the total structural response by defining a two-dimensional filtering process, both time and frequency domain, allowing the consideration of the time-varying contribution of ranks while denoising and mitigating errors due to numerical integrations. The process of implementing the wavelet-HT based-approach for filtering acceleration records is described below:

1) The DWT is performed over the relative acceleration records for each floor to obtain the rank decompositions $\ddot{x}_{i,k}$, where the index "*i*" represents the *i*th floor and "*k*", the *k*th rank.
2) The relative displacements for each rank $x_{i,k}$ are computed by the double integration of $\ddot{x}_{i,k}$ and removing any trend using a linear baseline correction.
3) The HT is performed over each floor and rank of the relative accelerations $\ddot{x}_{i,k}$ and relative displacements $x_{i,k}$ Eq. (1) shows the corresponding expressions for the relative accelerations $\ddot{x}_{i,k}$ of each floor and rank. The relative displacements $x_{i,k}$ are similarly obtained.

$$\ddot{x}_{i,k} = \mathrm{Re}\left[a_{i,k}(t)e^{\mathrm{j}\int \omega_{i,k}(t)dt}\right] \tag{1}$$

4) The Hilbert spectrum is performed over the top relative acceleration HSP_T using the rank decompositions as monocomponent sub-signals.
5) Step 4 is repeated for obtaining the Hilbert spectrum of the base total acceleration HSP_B.
6) The amplification between the input and output in the time-frequency domain is obtained by dividing the top and base Hilbert spectrums HSP_{TB}.
7) HSP_{TB} is graphically represented to identify the region dominated by the first mode response. This critical area is isolated by defining both low-pass and high-pass frequency envelopes.
8) The relative accelerations are filtered for each floor and rank, including the base, by reducing of all the instantaneous amplitudes $a_{i,k}(t)$ above the low-pass frequency and below the high-pass frequency envelopes using low-pass and high-pass filters.
9) The filtered relative accelerations are reconstructed from the reduced instantaneous amplitudes $a'_{i,k}(t)$ using Eq. (2), which yielded the first mode contribution ${}_1\ddot{x}_i$ of relative accelerations for each floor.

$${}_1\ddot{x}_i = \mathrm{Re}\left[\sum_{k=1}^{n} a'_{i,k}(t)e^{\mathrm{j}\int \omega_{i,k}(t)dt}\right] = \mathrm{Re}\left[\sum_{k=1}^{n} a'_{i,k}(t)e^{\mathrm{j}\theta_{i,k}(t)}\right] \tag{2}$$

10) Similar to steps 8 and 9, using the same envelopes defined in step 7, the relative displacements $x_{i,k}$ are filtered to obtain the first mode contribution ${}_1x_i$ of relative displacements.

After obtaining the relative responses ${}_1\ddot{x}_i$ and ${}_1x_i$, the methodology proposed by Kusunoki et al., (2018) is implemented to obtain the "tentative" representative responses A'_R and D'_R using Eqs. (3) and (4), respectively. Subsequently, the "tentative" capacity curve is extracted from the hysteretic relationship between A'_R and D'_R.

$$A'_R = \frac{\sum(m_i \cdot {}_1\ddot{x}_i)}{\sum m_i} + {}_1\ddot{x}_0 \tag{3}$$

$$D'_R = \frac{\sum(m_i \cdot {}_1x_i)}{\sum m_i} \tag{4}$$

4. RESULTS AND DISCUSSION

The most critical scenario (06story-record1), where the time-varying contribution of ranks issue is the most significant, is presented. This demonstrated the advantage of this 2D filter approach for the isolation of the first vibration mode from the total response. Figure 2 shows the transfer functions before and after the time history analysis corresponding to record one. Here, a high degree of nonlinearity is observed. The frequency of the second mode moved into the frequency range of the first mode (at the elastic stage). In this case, a frequency invariant-filter is no longer adequate. A variable frequency filter is essential for the analysis across the time domain.

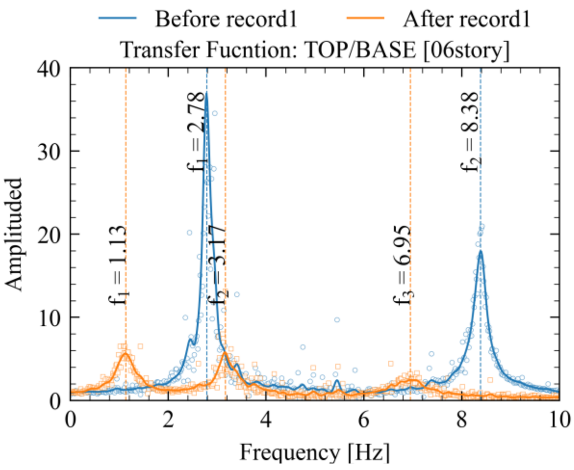

Figure 2. Transfer function; 06story-record1

Figure 3 shows the normalized top over base Hilbert spectrum and the envelope frequencies determined to achieve the accurate isolation of the first vibration mode. Using these frequencies envelopes and the representative instantaneous frequencies, the responses were filtered as is shown in Figure 4.

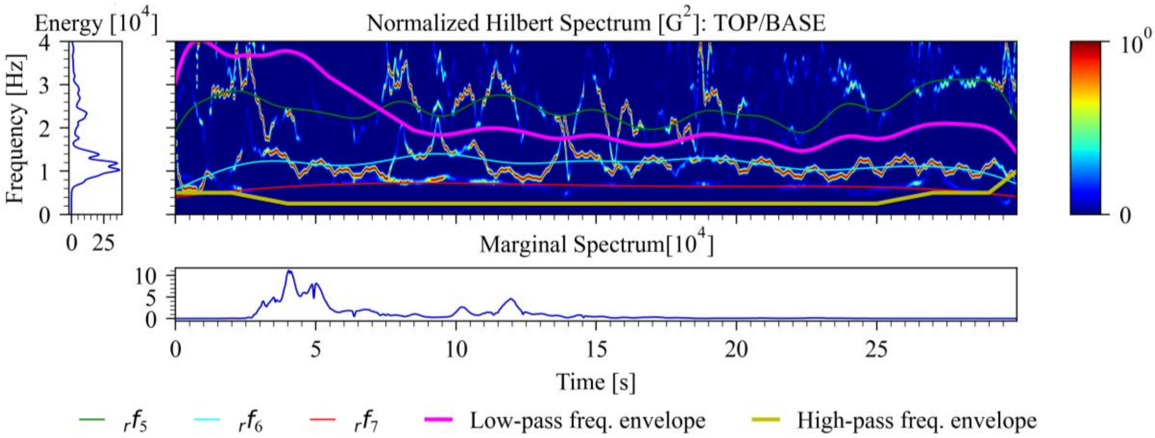

Figure 3. Low and high-pass frequency envelopes; 06story-record1

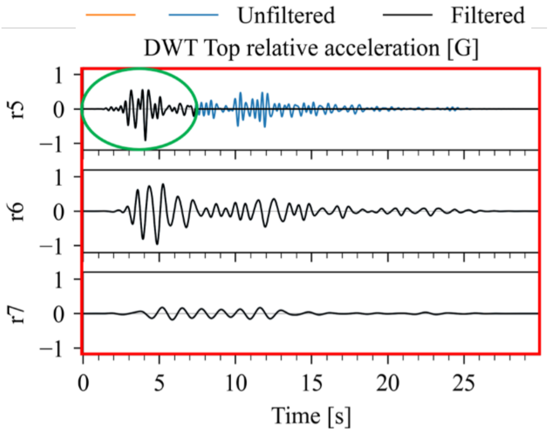

Figure 4. Filtered responses; 06story-record1

Here, the contribution of rank five is approximately until the 7 s, and lower ranks were completely removed. Using the base methodology, the selected ranks were 5, 6, and 7 (enclosed by the red square). However, this approach does not account for the varying contribution of rank 5; instead, it considers it in its entirety. Then, Eqs (3) and (4) were applied to both methodologies to obtain the tentative representative response of the equivalent SDOF and compared. Figure 5 shows the time range where the effect of nonlinearity was most pronounced (from 7 to 18 s). Here, the equivalent response obtained with the base method (constant boundary) was similar to the response of the unfiltered model, which shows that it still contains contributions from high modes, specifically from the second mode response. In contrast, the equivalent response obtained with the proposed methodology (varied boundary) successfully removed these higher mode contributions, effectively isolating the response of the first vibration mode.

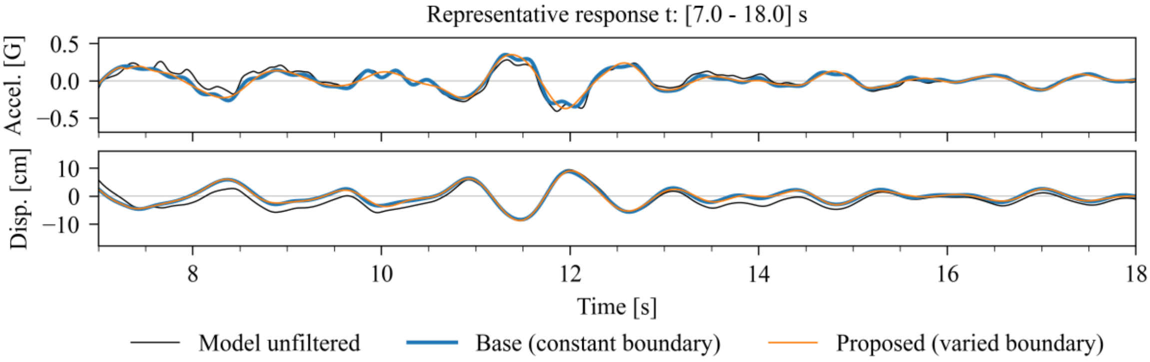

Figure 5. Filtered responses; 06story-record1

Figure 6 shows the transfer functions of the equivalent responses using both methodologies and compared with the original unfiltered response. It shows how the contributions of higher modes are entirely removed in both methodologies. However, in the frequency range corresponding to the second mode, particularly after the damage, the base methodology still exhibited residual contributions from the second mode response.

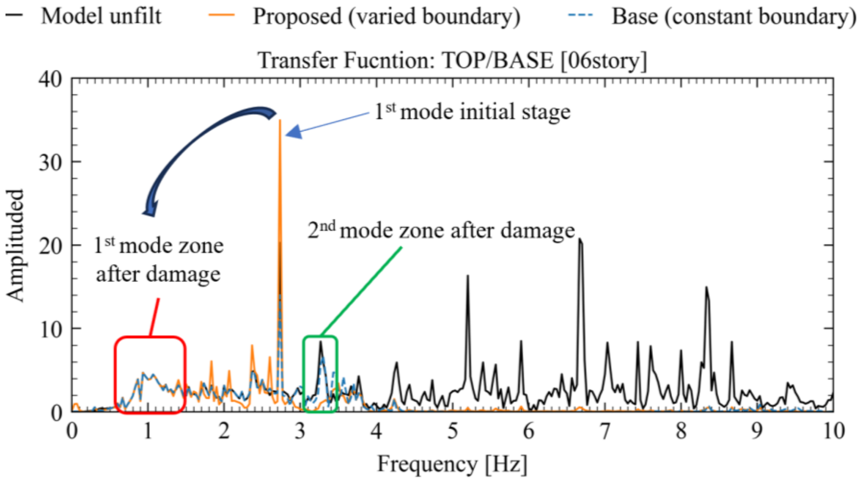

Figure 6. Transfer function after filter process; 06story-record1

Figure 7 shows the hysteretic relationship between the equivalent accelerations and displacements and the backbone curve extracted from this relationship. At first sight, the curves appear nearly identical, except for the region corresponding to lower displacements. In the time interval where the maximum response occurs, the selected ranks before the 7-s mark are identical in both methodologies.

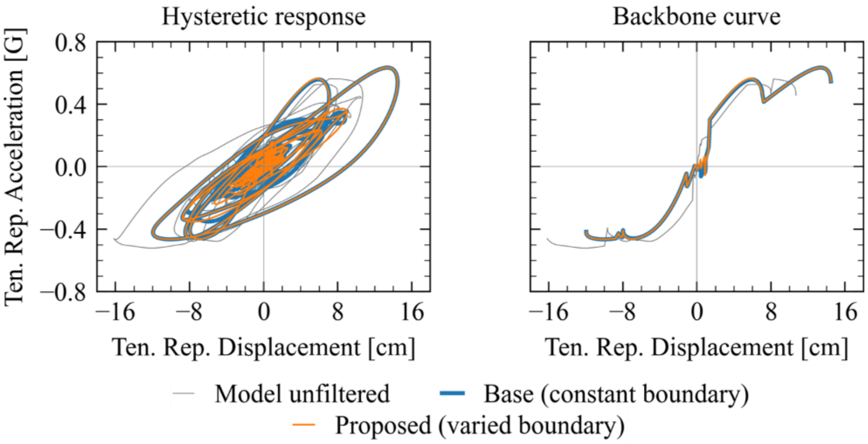

Figure 7. Capacity curve estimation; 06story-record1

Moreover, the backbone curve is complete at this stage because after the 7-s mark, no additional maximum responses add further points to the capacity curve.

In the critical time interval (from 7 to 18 s), both methodologies diverge considerably. In this time window, the proposed methodology discarded the contribution of rank five because it implements a varied frequency boundary, whereas the base methodology continues to include it. The time history of the representative acceleration in the base methodology exhibited local variations attributable to the residual contribution of the second mode. These variations manifest in the hysteretic response as concave and less-smooth shapes contrary to the proposed methodology. However, despite these differences in the hysteretic behavior, this segment does not add additional points to the backbone curve. This is because the maximum responses, which define the backbone curve, were already captured in the previous time window.

5. CONCLUSIONS

This methodology performs a dynamic filtering process in both time and frequency domains, which enables superior capability in isolating the fundamental mode response, particularly in structural systems with a high degree of nonlinearity where modal separation is challenging.

In most cases, the estimated backbone curves remained practically identical, even when the time-varying contribution of ranks occurred, primarily because the maximum responses defining these curves occurred before significant nonlinearity was introduced. This initial interval largely determined the curve's shape. After a critical point of nonlinearity, subsequent responses rarely exceed the initial maxima, leaving the capacity curve unmodified. Consequently, this phenomenon had minimal impact on the final capacity curve estimation.

Although the capacity curves obtained using both methodologies were practically identical, their hysteretic response curves differed considerably. In these curves, a substantial improvement was observed, particularly in the section that examined the influence of the contribution of second mode. This proposed methodology offers a significant advantage in enhancing the precision of hysteretic model estimation, which is crucial for constructing reliable mathematical models.

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